

## Philosophy of Mathematics (With reference to Plato, Aristotle, Kant, and Popper)

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### *Abstract*

*Philosophy of Mathematics starts with a simple question - What Mathematics is all about? What Mathematical numbers are? Unlike Physics, Mathematics primarily is not about matter and energy, it is not about the study of plants and animals like Biology. It is not linguistic in the ordinary sense of the word. It is not about society primarily like Sociology, Political science, or Anthropology. Great philosophers like Plato, Aristotle, Russell, Whitehead, Frege, and Popper have been trying to understand and make other people understand the essence of the Ontology of Mathematics. This article has attempted to understand the different and original ideas of great philosophers (Plato, Aristotle, Kant, and Popper) and try to interpret and share their views about the Ontology of mathematical numbers, theorems, equations, etc.*

**Keywords:** *Ontology, Epistemology, Plato, Aristotle, Kant, Popper, Mathematical Numbers.*

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### **Introduction**

Being a highly abstract discipline, it is always difficult to understand the proper subject matter of Mathematics. Normally Mathematics is supposed to be an important discipline of Science but unlike of Biology, Physics, and Chemistry (Physical Sciences), it is not about matter, energy, different characteristics of matter, or the physicality of animals and plants. The other most interesting characteristic of Math is that though it does not seem to be derived from the physical world, it is always easy and essential to apply it to the external and material world (Russell, 2003). Pythagoras of ancient Greece was not only a great mathematician but an outstanding thinker and a philosopher of religious and mystic nature influenced by the mysterious Ontology of Mathematics. For Pythagoras number is the ultimate reality of this universe (Stace, 2010). In the metaphysics of Pythagoras, the ultimate reality is abstract, idealistic, non-physical, and beyond this material world. Much influenced by the Mathematical philosophy of Pythagoras, Plato developed a wholly new and original system of philosophy. The philosophy of Plato starts with his epistemology (theory of knowledge) and according to this epistemology there is not a single world in front of us but two mysteriously interacting with each other (Stace, 2010). The one world is the world of material and physical things and the second world is their ideas of them. Plato rejects the common-sense point of view according to which the ideas of things are nothing but their general concepts of them. According to him, material things are temporary, transitory, momentary, and imperfect while ideas are perfect, absolute, and permanent. An imperfect thing cannot produce the perfect one. So according to this line of thinking there must be a world full of ideas beyond this time and space. According to Plato, Mathematical numbers are the inhabitants of the World of Ideas or Forms and not of the material world. Aristotle disagreed with

Plato denying the existence of the World of Forms altogether. According to Aristotle, the only world is, actually, the material world in front of us. Ideas of things according to Aristotle are the essence of them (Russell, 2003). The essence of material things cannot be separated from them having an independent existence of their own. Logically the question before Aristotle was what a mathematical number was. Aristotle considered numbers as an essential quality of material things. He was of the opinion that we could not consider any material object without justifying its quantitative nature (single, dual or plural). In this way, Aristotle tried to establish a relationship between material things and their ideas on one hand and material things and their numbers on the other. Are numbers, actually, the essential qualities of material things? The philosophical justification of numbers by Aristotle can easily be challenged. When anybody says 'two shoes' or 'one pair of shoes, he or she is saying the same thing. This simple example shows that though numbers can be applied to material things they cannot be considered the essential and necessary quality of a material thing.

The great philosopher Immanuel Kant tried also to explain the Ontology of Mathematics. For Kant Mathematics is all about the categories of mind (Kant, 1998). We can understand the explanation of Kant by taking a simple example. After perceiving a book in front of us (perceiving its color, shape, smell and solidity, etc.) one can conceive the same book into different categories.

- The book can be conceived as a unity.
- The same book can be conceived as a plurality (with reference to its pages etc.)
- This book can be conceived as an individuality (different from chairs, tables, benches, etc.).

Kant is of the opinion that mathematical numbers are neither in the material world nor they exist in the World of Forms as Plato presents. Logic and Mathematics have something to do with the categories of mind. For Karl Popper, Mathematics is the unintended effect and the by-product of the number series (1, 2, 3...). According to Popper, the number series is manmade but the existence of odd numbers, even numbers, and prime numbers is due to its unintended effect of it (Miller, 2010). Many scholars have criticized the explanation of Karl Popper because according to them neither Odd, Even, and Prime numbers nor the number series are manmade; the manmade thing is only the numerals (the name of the numbers in different languages) (Klemke, 1979). After presenting a brief introduction of the topic, it seems pertinent to discuss the above-mentioned concepts one by one.

## Research Objectives

The research objective of this study is to investigate the Ontology of Mathematics i.e.

- i. What is Mathematics all about?
- ii. Where does a Mathematical number exist?
- iii. What is the proper subject matter of this subject?

## Research Questions

The research questions are

- i. What is the individual uniqueness of Mathematics as a discipline?
- ii. What great philosophers have been saying about the Ontology of Mathematics?

## The explanation for the Ontology of Mathematics by Plato

Plato is of the opinion that mathematical numbers have an existence of their own, not in the material world and not in the subjective ideas of individuals. The numbers have their objective existence beyond time and

space. Temporal and Spatial things cannot be that perfect, absolute, universal, and incorrigible. Mathematical numbers, equations, theorems, etc. seem permanent, perfect, and universal. They cannot be derived from the spatial, temporal world. The metaphysics of Plato is called Objective Idealism in philosophy because the ideas or forms in the World of Ideas of Plato do not depend on any Mind or Minds (Russell, 2003). It is not difficult to understand that behind Plato's World of Forms there exists the Ontology of Mathematics and the mathematical philosophy of Pythagoras. In the humble opinion of the writer without the proper understanding of the nature of Mathematics, one cannot develop a theory like World of Ideas or World of Forms. The commonsense point of view does not accept the existence of ideas and forms separated from matter and mind. The common-sense point of view considers ideas of things as the general concepts of them and the common sense point of view does not accept ideas without the mind of Man or God. It was the understanding of Plato about Mathematics that leads to the magnificent metaphysics called World of Ideas or Forms in philosophy. For Plato mathematical ideas like any other ideas are innate in nature. Man is not born blind. Every human being has innate ideas or latent ideas at the time of his or her birth but these ideas also have an independent existence of their own beyond this time and space. For Plato in the World of Forms there exist many ideas in the forms of equations and theorems which are not been discovered by any human being yet. In other words, for Plato, mathematical knowledge is basically innate and not acquired. Human beings discover mathematical knowledge and not invent it (Popper, 1962).

### **Aristotle on Mathematics**

Aristotle was the student of Plato having a very different temperament. Aristotle is supposed to be the father of all Sciences. He was deeply interested in Biology (Zoology and Botany both), Logic, Political Philosophy, Practical Politics, Sociology etc. Actually, he was a man of practical nature. Plato on the other hand was a great poet, prose writer (dialogue writer), mathematician, and philosopher. Plato is more imaginative, creative, and artistic having great mathematical acumen. Aristotle denied the existence of any World of Ideas or World of Forms beyond any time and space. In this way, he denied the Dualistic Cosmology of Plato. According to him, there is only one world that is in front of us and we can perceive it through our sensuous experience. For him, numbers are the essential quality of material things (as mentioned above). There are many problems with its explanation of Aristotle. Not only human beings but animals do have sense organs. They can touch, smell, see, hear or have the sensations of taste but it is difficult to prove that they have the faculty of counting. Many animals recognize their babies through their smells and not through their numbers. Denying the existence of the World of Forms Aristotle actually denied the objective and independent existence of mathematical numbers. Numbers cannot be considered as essential qualities of material things though numbers can be applied to them. After rejecting the World of Forms of Plato, Aristotle gave the world of philosophy a concept of four causes (Ackrill, 1981). These causes were:

- Material cause
- Efficient cause
- Formal cause
- Final cause

Aristotle afterward reduced efficient and final cause to formal one and presented a concept of matter and form. In simple words, Aristotle's line of thinking does not accept any form without matter and any matter without form. The problem with the Ontology of Mathematics is that it does not seem to be explained in this way. The pure abstraction of Mathematics does not accept (it seems) the explanation given by Aristotle. In the paradigm of Aristotle, there is no room for pure abstraction except for the concept of God (which is matter less form according to him). In the paradigm of Aristotle Mathematical numbers seem unexplained and the concept of god seems inconsistent with the overall thinking process.

### **Kant on Mathematics**

The fundamental question for Kant as a philosopher was how to reconcile the traditions of Rationalism and Empiricism. According to Kant, epistemology seems insufficient without the creative synthesis of these two rival schools of thought and many important disciplines of human knowledge cannot be explained without this reconciliation. The big problem was, obviously, Mathematics. According to Kant Logic and Mathematics cannot be explained on empirical grounds. In order to explain the proper justification of different disciplines of human knowledge and in order to present before the world an epistemology that can explain much, he gave two creative concepts in this regard (Kant, 1998). They are:

- Sensibility (according to English translation).
- Understanding (according to English translation).

Sensibility has something to do with human perception and Understanding has strong connections with human conceptions. Kant is of the opinion that the faculty of Sensibility regulates the sense data and the faculty of Understanding conceives the same perceived object into different categories. For Kant Mathematics and Logic are all about the categories of mind. The problem with this explanation of Kant is that whole Mathematics becomes mental and subjective in this way (Russell, 2003). In the 20<sup>th</sup> century Russell (along with Whitehead) and Frege in Germany tried to present a sound explanation for the Ontology of Mathematics. Interestingly all of them rejected not only Aristotle and John Stuart Mill (who tried to explain Mathematics on empirical grounds) but also Kant and came very close to the explanation given by Plato (Russell, 2009). Popper (1902-1994) gave a concept of 'World 3' trying to replace the World of Forms of Plato. Popper also gave an explanation of mathematical numbers.

### **Popper on Mathematics**

Karl R. Popper was a great philosopher of the 20<sup>th</sup> century. His philosophical journey started with his distinction between science and pseudo-science. After presenting this creative distinction he moved towards Epistemology, Ethics, and Political Philosophy. In the later years of his life, he presented the concept of 'World 3' which is basically a pluralistic concept of the world (Popper, 1992). 'World 3' is actually a concept or an effort to replace the World of Forms of Plato. For Plato there are two Worlds - World 1 is the world of material objects and World 2 is the world of ideas. For Popper, there are three worlds. 'World 1' is the world of material objects, 'World 2' is the world of mental states of individuals, and 'World 3' is the world of all possible human creations. After rejecting the World of Forms of Plato Popper also tried to give an explanation for the Ontology of Maths. For Popper the number series (1, 2, 3...) is manmade but the distinction among Odd, Even and Prime numbers are the unintended effects and the by-product of this series (Cambier, 2016). The explanation given by Popper does not seem very sound. E. D. Klemke is of the opinion that neither number series nor Odd, Even, or Prime numbers can be considered as a human creation. The human creation is only the numerals and not numbers. The objection of Kelmke seems valid. The distinction between manmade things and their unintended effects seems quite arbitrary. It is very difficult to consider all the human knowledge as manmade and it is quite inconsistent to consider the unintended effect as a human product in a strange way (man-made yet independent of all human individuals).

### **Conclusion**

In the history of Western philosophy, the Ontology of Mathematics has always been a difficult subject. The reason is quite obvious. Mathematics is neither testable nor linguistic. Mathematical numbers are extremely abstract. Plato is of the view that mathematical numbers have their own independent existence in the World of Forms or Ideas. This world of Plato is abstract, perfect, and beyond time and space. For Aristotle matter cannot exist without form and form cannot exist without matter. Mathematical numbers cannot have their independent existence and they must be the essential qualities of material objects. The great philosopher

Kant also tried to explain Mathematics according to his own Transcendental Idealism. In 20<sup>th</sup> century John Stuart Mill also tried to give Mathematics an empirical ground but Frege rejected it by giving his own explanation. Interestingly the explanations given by Russell, Whitehead and Frege came very close to the views of Plato. That is why Whitehead considered the whole philosophy after as the footnotes of it (Popper, 1967). Popper in the later period of his life tried to replace the World of Forms of Plato with his 'World 3'. He rejected the Monism of Idealism and Materialism and the Dualism of Plato and Descartes. He presented a pluralistic concept of the world. For him Mathematics is a manmade product but after its coming into existence it liberates itself and becomes independent of the minds of human beings. In the humble opinion of the writer the explanation of Plato for the Ontology of Mathematics is outstanding and it can be called better than the best.

## Recommendations

The researcher proposes the following recommendations of the study:

- i. There is a need for further research in order to get satisfactory answers about the nature of Mathematics.
- ii. Not only mathematicians but philosophers also should attempt again and again in order to solve the mysteries of Maths.

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